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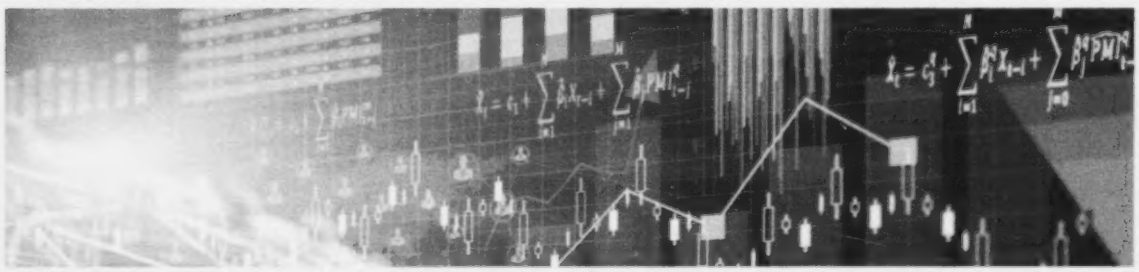
Information Sharing and Bargaining in Buyer-Seller Networks



by Sofia Priazhkina and Frank H. Page

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Information Sharing and Bargaining in Buyer-Seller Networks

by

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Abstract

This paper presents a model of strategic buyer-seller networks with information exchange between sellers. Prior to engaging in bargaining with buyers, sellers can share access to buyers for a negotiated transfer. We study how this information exchange affects overall market prices, volumes and welfare, given different initial market conditions and information sharing rules. In markets with homogeneous traders, sharing always increases total trade volume. The market reaches Walrasian trade volume when there are more buyers than sellers or when sellers have more bargaining power. In most cases, market surplus is completely reallocated to sellers. In the markets with heterogeneous traders, sharing may either increase or decrease total trade volume. When sellers have more bargaining power than buyers, information exchange leads to trade above the Walrasian level, thus leaving inefficiency only due to overproduction of high-cost sellers. As a result of information sharing, the buyers who value goods the least will be squeezed out from the market independent of their location and bargaining power. We also show that if, together with information exchange, sellers assign property rights on the information, exchange leads to lower volume and market prices than exchange without property rights.

Bank topics: Economic models; Firm dynamics; Market structure and pricing
JEL codes: D4; D43; D8; D21; D85; C71; C78; L1; L13

Résumé

Nous présentons un modèle de formation de réseaux stratégiques acheteurs-vendeurs dans lequel les vendeurs s'échangent de l'information. Avant d'entamer la négociation avec les acheteurs, les vendeurs peuvent se donner mutuellement accès à des acheteurs moyennant un paiement de transfert négocié. Nous étudions les effets de cet échange d'information sur l'ensemble des prix de marché, les volumes et le bien-être, compte tenu de diverses conditions de marché initiales et règles de partage de l'information. Sur les marchés où les opérateurs sont homogènes, l'échange d'information accroît systématiquement le volume total des échanges commerciaux. L'équilibre walrasien est atteint sur le marché lorsque les vendeurs sont plus nombreux que les acheteurs ou lorsqu'ils ont un plus grand pouvoir de négociation que ces derniers. Dans la plupart des cas, l'excédent commercial revient entièrement aux vendeurs. En situation d'hétérogénéité des opérateurs, l'échange d'information peut accroître ou abaisser le volume total des échanges. Quand les vendeurs ont un pouvoir de négociation supérieur à

celui des acheteurs, l'échange d'information se traduit par des volumes supérieurs au niveau d'équilibre walrasien, et l'inefficience du marché ne peut alors résulter que de la surproduction des vendeurs ayant des coûts de production élevés. Du fait de cet échange d'information, les acheteurs qui accordent le moins de valeur aux biens seront évincés du marché, quel que soit le lieu où ils se trouvent ou leur pouvoir de négociation. Nous montrons en outre que si les vendeurs non seulement échangent de l'information, mais encore protègent cette information par des droits de propriété, l'échange aboutit à des volumes d'échanges commerciaux et à des prix plus bas qu'en l'absence de droits de propriété.

Sujets : Modèles économiques; Dynamique des entreprises; Structure de marché et fixation des prix
Codes JEL : D4; D43; D8; D21; D85; C71; C78; L1; L13

Non-Technical Summary

In this era of information technology, it is difficult to underestimate the effect that market information has on prices and trades of a particular seller. To gain more market power and potentially increase market volumes and trades, sellers often buy information about their current and potential buyers from other sellers. This information exchange among sellers and its impact on the market is the main focus of this paper. In particular, we consider information exchange among sellers as a two-period network formation game with buyers and sellers acting strategically. We model initial information frictions by assuming that sellers and buyers can trade only if they are connected in a bipartite graph of business relationships. Before sellers trade with buyers, they can strategically share access to the buyers for negotiated side payments. This information exchange reduces the number of frictions in the economy but may also reduce welfare for certain buyers and sellers. We study how this information exchange affects overall market prices, volumes, and welfare of different sellers and buyers given various initial market conditions and information sharing rules.

Depending on the initial market structure and preferences of buyers and sellers, information exchange may lead to very different market outcomes: total market trade volume can either increase, decrease, or stay the same, as can prices. Nevertheless, it is still possible to characterize the general impact of one-sided information exchange on overall market performance. We show that, in markets with identical sellers and buyers, information sharing always increases total trade volume. It is often the case that some sellers abstain from sharing market information to keep higher prices in their local markets. However, when sellers are in a more preferable position than buyers—that is, when the number of buyers exceeds the number of sellers or when sellers have greater bargaining power than buyers—total trade volume reaches maximum. In most cases, market surplus is completely reallocated to sellers as they are the only ones sharing information. Buyers can benefit from information sharing only in the special case when the profit of sellers from matching non-trading players covers the loss from dissolved monopoly rents.

In markets with heterogeneous traders, sellers may become intermediaries not only to increase prices and volumes, but also to reduce production costs. As such, sharing may lead to an increase or a decrease of total market volume. We prove that when sellers have more bargaining power than buyers, information exchange leads to trade above the Walrasian level. It means that market inefficiency can only arise because of overproduction of high-cost sellers. In the heterogeneous case, we also shed light on the general question of when and how the information exchange among sellers may benefit buyers: if sellers have more bargaining power than buyers, even though information sharing may lead to unfavorable prices for buyers, the goods will be reallocated to buyers who value the good the most. In other cases, a weaker result takes place, and trading buyers are twice less likely to be the highest bidders.

In the last section of the paper, we consider information sharing with transferable rights as an alternative to the non-transferable rights case discussed so far. We show that, if sellers exchange information and assign property rights to the information, the referrals lead to lower volume and market prices than referrals without property rights.

1. Introduction

What is the effect of information sharing among sellers about buyers on prices and trade volumes? As the number of business-to-business transactions increases, the role of information sharing raises interesting questions about its impact on market outcomes. We answer this question using bipartite networks, where nodes represent buyers and sellers and links represent market access. Before sellers trade with buyers, they can strategically share access to the buyers with one another for negotiated side payments.[2] This information exchange changes the initial network structure and affects the trade choices that sellers and buyers make after the information is exchanged. We shed light on the resulting market structures and find the corresponding effect of referrals on market trades. We also compare the stable outcomes with the Walrasian outcome and determine how the trade surplus is divided between sellers and buyers. In the last section, we change the information sharing rules to show the effect of information property rights on market outcomes.

The economic intuition of our model can be gained from real-world examples. First, consider a simple case of a local bazaar or retail market where a seller can make an immediate sale but realizes that he is out of stock. This seller runs to another seller "next door" and a minute later brings the required product to the buyer. In the end, the buyer purchases the product and leaves satisfied while the payoff from the sale is split between the two sellers. In this example, the process of information exchange is equivalent to the process of gaining access to a buyer. As such, information exchange leads to a trade that would have otherwise not occured.

There are other examples of sharing agreements that lead to higher trade volumes. Doctors, lawyers and other professionals who have limited labor hours and face unequal demand may use information sharing in the form of referrals. For example, a popular practitioner may pass his patients to a less popular colleague free of charge or, if the referral becomes regular practice, allow the colleague to see the patients in the same building for a negotiated payment. In retailing, information sharing often takes place in the form of platforms. Large stores, such as Amazon.com, Macy's, and Kroger, sell only a limited volume of their own brands and make the majority of their profits by serving as platforms for other competing brands. In fact, providing access to buyers for smaller brands becomes the main business function of these stores. In a market of Internet sales, one company may share email addresses of potential clients or exchange information contained in clients' cookies.[3]

Information sharing among sellers can also lead to an increase in prices. An example can be found in the labor market. Consider two job seekers with equal productivity. The first one has more valuable connections with employers and, therefore, can secure more valuable job offers. Although both job seekers are competing in the same market, the first still decides to inform the other about the vacancy he is not interested in. This information exchange may lead to a better job for the second job seeker either directly, if he accepts the offer

[2] We do not consider the case of buyers exchanging information in part because, in most real-life markets, sellers tend to possess significantly more information and use more resources to manage the information.

[3] Abraham et al. (2013) [1] verifies that cookies create substantial information asymmetries among sellers.

from the new employer, or indirectly, if he uses it in negotiation with other employers.

Finally, consider the example of firms in a geographically segregated market. Depending on the production costs of firms and the values of consumers, information sharing between a firm in a developed country and a firm in a developing country can lead to different outcomes. For instance, assume a firm in a developed country outsources all orders to a firm in a developing country to decrease production costs. As a result, the firm in the developing market reorients itself towards export instead of the domestic market. As such, market sharing decreases the total volume of trade but still increases profits for both firms, provided buyers from the developed country pay high enough prices. Alternatively, market sharing between the two firms can occur because one firm reaches its production capacity and uses another firm to expand. In this case, total trade volume may increase, given that the firms will not reduce their production levels.

The example of geographically segregated markets illustrates how variations in the initial market conditions can lead to very different market outcomes. Depending on the production costs of firms and the values of consumers in both countries, total trade volume can either increase, decrease or stay the same, as can prices. Moreover, there can be multiple referrals for each sale if the developing market firm also delegates the tasks to smaller firms or freelancers. To capture this complexity in our model, we account for a wide range of initial network structures with different bargaining specifics and two types of information sharing rules.

The results of the paper are as follows. We show that the effect of information sharing depends on the initial network structure, bargaining power of sellers relative to buyers, the distribution of production costs of sellers and the consumption values of buyers. In markets with homogeneous traders, sellers share market information for two reasons: increase in price and new trades. In particular, sellers that would trade even without the information exchange want to expand their markets to increase prices by increasing competition among their buyers. At the same time, initially non-trading sellers buy market access to increase their trade volumes. We show that information sharing among homogeneous sellers always increases the total trade volume. Some sellers may abstain from giving away their non-trading buyers to keep higher competition and prices in their local markets. However, when sellers are in a more preferable position than buyers—that is, when the number of buyers exceeds the number of sellers or when sellers have more bargaining power than buyers—total trade volume reaches Walrasian trade volume. In most cases, market surplus is completely reallocated to sellers as they are the only ones sharing information. However, in some cases, information exchange leads to lower market prices, which also benefits buyers. We determine that buyers can benefit from information sharing only in markets with more sellers than buyers and with sellers having more bargaining power. When sellers have more bargaining power, they may be willing to decrease their prices in favor of intermediation profits by sharing their market access with non-trading sellers. This happens because the profit from matching non-trading players covers the loss from dissolved monopoly rents.

In markets with heterogeneous traders, sellers may become intermediaries not only to increase prices and volumes, but also to reduce production costs (as in the example with geographically segregated firms). In the heterogeneous case, sharing may lead to an increase or a decrease of total market volume. We prove that when

sellers have more bargaining power than buyers, information exchange leads to trade above the Walrasian level. It means that market inefficiency can arise only because of overproduction of high-cost sellers. In the heterogeneous case, we also shed light on the general question of when and how the information exchange among sellers may benefit buyers. One of our main theorems claims that, if sellers have more bargaining power than buyers, the goods are allocated to those buyers who place the highest value on the good. This means that, even though information sharing may lead to unfavorable prices for buyers, the goods will be reallocated to buyers who value the good the most. In the other case—when buyers have more bargaining power than sellers—the results are weaker: given k trades, the trading buyers belong to the set of $2k$ buyers with the highest values. In this case, it is not always true that the trading buyers value goods the most, because sellers may intentionally leave high-valued buyers hidden to bid the prices higher.

In the last section of the paper, we use a market setup with homogeneous traders to show the effect of information property rights on the information exchange. Also, we consider information sharing with transferable rights as an alternative to the non-transferable rights case discussed so far. This means that a seller takes responsibility to cease trade with buyers once he reveals the information about these buyers to other sellers. We show that if sellers exchange information and assign property rights on the information, the referrals lead to lower volume and market prices than referrals without property rights.

The paper has the following structure: Section 2 provides a literature review; Section 3 formally defines the set of feasible networks and rules of the game; Section 4 presents notions of stability; Section 5 and Section 6 contain solutions of the model for homogeneous and heterogeneous traders, respectively; Section 7 considers the network formation game with alternative information property rights; Section 8 summarizes the paper; and the Appendixes contain the proofs.

2. Literature review

The research question raised in this paper contributes to the literature on referrals and information sharing between businesses. With the development of information markets, the referral business model has become more popular among intermediaries and firms. Condorelli, Galeotti and Skreta (2013)[10] showed that the referral business scheme is preferred by an intermediary to the process of buying and reselling. This process helps sellers to avoid search costs, while at the same time it may increase prices because of its collusive nature. Arbatskaya and Konishi (2012)[2] provide conditions where referrals are beneficial for both sellers and buyers in the non-network setup. We consider the referral process in the markets with a network structure and allow any seller to provide a referral. Lippert and Spagnollo (2010)[24] also emphasize the importance of considering information flows and action choices together in network markets. Literature on platform competition in two-sided markets started by Rochet and Tirole (2003)[30] is also relevant to our research question as it deals with multiple hierarchies of sellers.

The process of referrals is more generally related to the literature on intermediaries and evolving trade networks. For example, Blume et al. (2009)[8] consider a bargaining network with intermediaries. The

intuition behind their model is similar to the intuition underlying our model; however, our assumptions are different. In our model, a seller chooses whether or not to be an intermediary and the intermediary's fee is endogenously determined. Belleflamme and Bloch (2004)[3] also explore oligopoly markets on networks with sharing agreements by which firms commit not to enter each other's territory. In the last section of our paper, our model allows sellers to share the markets and exchange information at the same time. We then focus on the changes in prices and quantities that result from these interactions.

The theoretical approach used in the paper comes from the literature on network formation games with bargaining. Notions of stability in network formation games were first discussed in Jackson and Wolinsky (1996)[22] and then extended by many researchers (see Bloch and Jackson (2006)[5] and Dutta and Bloch (2011)[4] for the recent review of network formation games). Here, we use an extended notion of pairwise stability with transfers based on Bloch and Jackson (2007)[6] together with the notion of stable sets based on Page and Wooders (2007, 2009)[28, 27].

In the homogenous case of the model, we represent a network as a composition of subnetworks of three different types similar to Corominas-Bosch (2004). The work of Elliot (2013)[14] proposes a complementary research to Corominas-Bosch (2004)[11] and Kranton and Minehart (2000)[23] for the case of heterogeneous traders. We relate the static bargaining mechanism to the dynamic mechanism of Corominas-Bosch (2004)[11] when traders are homogeneous, and to stable networks of Elliot (2013)[14] when traders are heterogeneous.

In the literature on bargaining networks, the central issue is the details of the bargaining mechanism. Several papers propose different bargaining protocols, including Bertrand competition (Guzman (2011)[19]), Cournot competition (Goyal and Joshi (2006)[17]), ascending-bid auction (Kranton and Minehart (2000)[23]), alternating-offer bargaining (Corominas-Bosch (2004)[11]), and bilateral negotiations similar to Rubinstein (1982)[31] mechanism (Polanski (2007)[29], Even-dar et al. (2007)[16]).[4] Manea (2011)[25] builds an infinite horizon model of a bargaining game. The idea of contingent contract in the bargaining networks is also captured by Mauleon et al. (2011)[26].

3. Model

3.1. Market structure

The market is populated by two types of traders: sellers $S = (s_1, s_2, s_3, ..., s_n)$ and buyers $B = (b_1, b_2, b_3, ..., b_m)$. Each trader trades either one or zero units of a good. Sellers have heterogenous production cost values $(v(s_1), ..., v(s_n))$ and buyers have heterogenous consumption values $(v(b_1), ..., v(b_m))$.

The market experiences information frictions, such that traders can only trade with the counterparties they know. We use a bipartite network to represent the trade contacts between sellers and buyers. If traders are matched according to some network G,

[4] Some papers focus on the bargaining process with a sequence of proposals following some exogenously determined order (see for example Currarini and Morelli (2003)[12]). The sequential approach has its advantages, such as uniqueness of equilibrium, as well as disadvantages, including the dependence of equilibria on the exogenously determined mechanism.

$$G = \{g_{ij} \in \{0,1\},\ s_i \in S,\ b_j \in B\},$$

each seller $s_i \in S$ can trade only with the buyers (nodes) it is connected to

$$N(s_i, G) = \{b_k \in B,\ \text{s.t. } g_{ik} = 1\},$$

and each buyer $b_j \in B$ can only trade with the sellers (nodes) it is connected to

$$N(b_j, G) = \{s_k \in S,\ \text{s.t. } g_{kj} = 1\}.$$

In other words, connections $N(s_i, G)$ determine the local market for seller s_i and connections $N(b_j, G)$ determine the local market for buyer b_j.

3.2. Timeline

We want to know how market trade surplus, prices, and quantities change when sellers are allowed to share their trade connections with each other. For this purpose, we consider the following two-period model:

- Ex-ante, traders are connected via status quo network G_0.
- At $t = 1$, sellers strategically update network connections by selling the information about their trade contacts to each other. This results in the upgrade of the status quo network G_0 to sharing network G_{sh}.
- At $t = 2$, given the market structure G_{sh} established at time $t = 1$, sellers and buyers trade goods with each other.

3.3. Time $t = 1$: network formation

In the first stage of the game, sellers upgrade network G_0 to network G_{sh} by selling contact information. We will refer to this process as information sharing and market expansion.

The mechanism of information sharing unravels in a similar way to Bloch and Jackson (2007)[6]. All sellers simultaneously announce their offers to all other sellers, and offers clear. Without loss of generality, consider two sellers s_i and s_j. At $t = 1$, seller s_i announces a set of buyers to share with s_j

$$L_{i,j} \subset N(s_i, G_0) \cup N(s_j, G_0)$$

and the corresponding transfer $t^s_{i,j}$ from s_i to s_j. At the same time, seller s_j announces a set of buyers to share with s_i

$$L_{j,i} \subset N(s_i, G_0) \cup N(s_j, G_0)$$

and the corresponding transfer $t^s_{j,i}$ from s_j to s_i. If two sellers share access to multiple buyers, all information they share is priced in a single contract. The sharing agreement takes place if and only if the proposals of both sellers are identical and both sellers agree on the transfer:

$$L_{i,j} = L_{j,i} \neq \varnothing,$$

$$t^s_{i,j} + t^s_{j,i} = 0.$$

If the sharing agreement is made, we mark the positive transfer from s_i to s_j

$$t_{i,j} = \max(t^s_{i,j} - t^s_{j,i}, 0).$$

In case the agreement is not made, we mark $t_{i,j} = 0$.

Information sharing upgrades status quo network G_0 to sharing network G_{sh} according to the prespecified network formation rules. We consider two different regimes of network formation rules. Under the regime with non-transferable information rights, a referral does not restrict the seller from dealing with the previously connected buyers. Under the regime with transferable rights, a seller must "forget" a buyer after he sells information about this buyer to another seller. It is convenient to think about these contracts as exclusive supply contracts, such that orders from one seller are redirected to another seller for a certain fee.

Sharing with non-transferable rights. *Information sharing between s_i and s_j leads to formation of network connections between seller s_i and buyers $N(s_j, G_0) \cap L_{i,j}$ and connections between seller s_j and buyers $N(s_i, G_0) \cap L_{i,j}$.*

Sharing with transferable rights. *Information sharing between s_i and s_j leads to a) formation of network connections between seller s_i and buyers $N(s_j, G_0) \cap L_{i,j}$ and connections between seller s_j and buyers $N(s_i, G_0) \cap L_{i,j}$; b) displacement of network connections between seller s_i and buyers $N(s_i, G_0) \cap L_{i,j}$ and connections between seller s_j and buyers $N(s_j, G_0) \cap L_{i,j}$.*

In Sections 3 to 6, we consider only sharing with non-transferable information rights. In Section 7, we extend some of our results to the transferable rights regime to observe the effects of information property rights on stability and efficiency of the market outcome.

3.4. Time $t = 2$: trades on the network

Given updated network G_{sh}, sellers and buyers form trade agreements. Each seller s_i submits ask price $p^s_i \geq v(s_i)$ and chooses the counterparty buyer $l^s_i \in N(s_i, G_{sh})$, and each buyer b_j submits bid price $p^b_j \leq v(b_j)$ and chooses the counterparty seller $l^b_j \in N(b_j, G_{sh})$. A trade agreement is formed between s_i and b_j if and only if they make simultaneous offers to each other and agree on the price:

$$l^s_i = b_j \text{ and } l^b_j = s_i,$$

$$p^s_i = p^b_j.$$

If the trade agreement is signed, we mark quantity $q_{i,j} = 1$ and price $p_{i,j} = p^s_i = p^b_j$; otherwise we mark $q_{i,j} = p_{i,j} = 0$.

In this paper, we also assume that sellers are not allowed to collude on prices given any network formation rules:

Assumption 1. *At $t = 1$, sellers cannot commit to a specific trade behavior at $t = 2$.*

As an example of collusion with non-transferable information property rights, two sellers s_1 and s_2 could agree that s_1 would not sell his good—creating excess market demand and raising prices—and that s_2 would then share his (artificially elevated) profits with s_1. We do not allow for this type of market manipulation. We also do not allow sellers to cooperatively increase prices. However, we allow for s_1 to shut down his production in favor of s_2 if this market outcome results naturally from market competition and buyers would not buy from s_1 even if s_1 produced the good. With transferable information property rights, sellers would still not be able to collude on prices. However, we allow for them to collude on market quantities when one seller redirects orders to another seller.

3.5. Defaults

A seller defaults at time $t = 2$ if total cost exceeds total revenue. Although sellers do not default in equilibrium, the procedure of default should be defined properly to characterize off-the-equilibrium paths. To see this, consider an example where seller s_1 shares information about buyer b_1 with seller s_2 in exchange for transfer $t_{2,1} > 0$. If seller s_1 sells the same information to another seller s_3, the competition for buyer b_1 will increase, which may decrease the revenue of s_2 sufficiently enough for s_2 to default. Defaulted s_2 will pay only a fraction of transfer $t_{2,1}$ to seller s_1. Therefore, it is important to define how s_2 repays transfer in the case of default in order to determine the incentives of s_1 to deviate from the equilibrium.

We assume pro rata default rule everywhere in the paper with the exception of Appendix E, where we relax this assumption to show that information sale can be sequential.

Assumption 2. *If seller s_i defaults, he is obligated to repay his debt $(t_{i,1}, ..., t_{i,n})$ to $(s_1, ..., s_n)$ on a pro rata basis using all available revenue from trade and information sharing.*

Assumption 2 means amounts $(F_{i,1}, ..., F_{i,n})$ that s_i needs to repay to $(s_1, ..., s_n)$ in case of default are proportional to amounts $(t_{i,1}, ..., t_{i,n})$.

Given sharing and trade agreements, the default status of seller s_i is captured by the default indicator function:

$$D_i = \begin{cases} 1, & \text{if } \sum_{j=1}^{m}(p_{i,j} - v(s_i))q_{i,j} + \sum_{k=1}^{n}(t_{i,k} + t_{k,i}(1 - D_k) + F_{k,i}D_k) < 0 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

All default indicators $(D_1, ..., D_n)$ are interdependent and can be found by solving the system of equations of type (1). This system of equations may have multiple solutions. The following theorem claims that a problem of multiplicity arises only when positive transfers between sellers form cycles (e.g., $t_{ij} > 0$, $t_{jk} > 0$, $t_{ki} > 0$ form a cycle for sellers s_i, s_j, and s_k).[5]

[5]This result is similar to the uniqueness result of Eisenberg and Noe (2001)[13] and Elliott et al. (2014)[15]). Defaults in networks with cliques are also considered by Blume et al. (2011)[7].

Lemma 1. *The set of all vectors $(D_1, ..., D_n)$ that satisfies (1) for $i = 1, ..., n$ forms a complete lattice with maximum and mini -a elements. Moreover, if positive transfers $\{t_{i,j}\}_{i,j}$ do not form cycles, the set of default indicators is uniquely determined.*

Proof. The proof of Lemma 1, together with other proofs, appears in Appendixes. □

As a result, the problem of multiplicity does not arise in the equilibrium, as it is shown further in the paper.

3.6. Payoffs

Given trade offers and sharing offers, the utility of solvent seller s_i is defined as the profit from both trade and information exchange:

$$U(s_i) = \sum_{j=1}^{m}(p_{i,j} - v(s_i))q_{i,j} + \sum_{k=1}^{n}(t_{i,k} + t_{k,i}(1 - D_k) + F_{k,i}D_k), \tag{2}$$

and the utility of buyer b_j is defined as the profit from trade:

$$U(b_j) = \sum_{i=1}^{n}(v(b_j) - p_{i,j})q_{i,j}. \tag{3}$$

The utility of a defaulted seller is assumed to be negative infinity, which means that any seller prefers autarky to default.

4. Notions of stability in networks

We use a concept of subgame perfect equilibrium with pairwise stable contracts. The equilibrium notion is defined using backward induction. First, a subgame at $t = 2$ is considered.

Definition 1. Given network G_{sh}, a set of trade offers and corresponding trades are pairwise stable if a) each trading player is weakly worse off by not trading, and b) any deviation by a buyer-seller coalition makes at least one member of the coalition strictly worse off or leaves both members indifferent.

If seller s_i and buyer b_j trade at price $p_{i,j} = p_i^s = p_j^b$, they split the surplus of size

$$\alpha_{i,j} = v(b_j) - v(s_i) \tag{4}$$

such that seller s_i gains $p_{i,j} - v(s_i)$ and buyer b_j gains $v(b_j) - p_{i,j}$. It is intuitively clear that stable trades in our game correspond to the stable outcomes in the assignment game of Shapley and Shubik (1971)[32]. We apply the assignment game results to our model. First, the set of stable trades is non-empty and equivalent to the core of the assignment game. Second, in the games with degenerate value functions, all stable trade offers generate the same profits for buyers and sellers. Third, for each stable assignment, the set of stable prices forms a complete lattice. These observations make the equilibrium selection straightforward. Given maximum matching, we select the equilibrium prices as follows:

Assumption 3. *Among all pairwise stable trade prices* $[\underline{p_{i,j}}, \overline{p_{i,j}}]$ *the price that emerges is*

$$p_{i,j} = (1-z)\underline{p_{i,j}} + z\overline{p_{i,j}},$$

where $z \in (0,1)$ *is an exogenous parameter that measures the bargaining power of one seller against one buyer.*

If we fix the equilibrium pairwise surplus $\alpha_{ij} > 0$, the stable payoff that seller s_i can get trading with b_j belongs to some interval $[\underline{u}(s_j), \alpha_{ij} - \underline{u}(b_j)]$, and the payoff that buyer b_j can get belongs to some interval $[\underline{u}(b_j), \alpha_{ij} - \underline{u}(s_i)]$, where $\underline{u}(\cdot)$ determines the minimum stable payoff given market trade volumes. According to Assumption 3, profits from trade of seller $s_i \in S$ and buyer $b_j \in B$ are correspondingly determined as

$$u(s_i) = p_{i,j} - v(s_i) = \underline{u}(s_i) + z(\alpha_{i,j} - \underline{u}(s_i) - \underline{u}(b_j)), \tag{5}$$

$$u(b_j) = v(b_j) - p_{i,j} = \underline{u}(b_j) + (1-z)(\alpha_{ij} - \underline{u}(s_i) - \underline{u}(b_j)). \tag{6}$$

Given pairwise stable trades in all bargaining subgames, we define pairwise stable contracts for the whole game:

Definition 2. The game payoffs and strategies form a subgame perfect equilibrium with pairwise Nash stable contracts (SPPNS) when

1) trade offers are pairwise stable in each subgame at time $t = 2$;

2) transfers are pairwise (Nash) stable at $t = 1$, which means that

a) each seller is weakly worse off by breaching one or multiple sharing contracts, and

b) any deviation by a seller-seller coalition, including breaching existing agreements and forming a new sharing agreement with each other, makes at least one member of the coalition strictly worse off or leaves both members indifferent.

In this paper, the cost of communication among sellers as well as settlement costs are assumed to be zero. However, we say that among all networks that provide the same payoffs, we pick the one that contains the smallest number of links. In other words, if removal of a link does not change the profits of all players, we assume that this link is not formed. If multiple equilibria exist, we pick the equilibria that provide the maximum number of trades to characterize prices and quantities.

Although the notion of pairwise stability (Jackson and Wolinsky (1996)) is widely used in the network literature, the existence of pairwise stable transfers is not guaranteed. In our model, pairwise transfers at $t = 1$ fail to exist under some parameters. Nevertheless, even when a pairwise stable network does not exist, it is still possible to characterize market prices and quantities by using the concept of basins of attraction (Page and Wooders (2009)[27]). Basins of attraction is a generalization of the pairwise stable equilibrium. The basin of attraction is defined as a set of market outcomes that can be reached but never escaped from

as a result of sequential strategic deviations. Each deviation of this kind should weakly benefit the deviating coalition. The sequences of single and pairwise strategic deviations are called (weakly) improving paths.

Definition 3. *A set of networks* $\mathbf{G_{sh}}$ *is called a basin of attraction if from any network* $G_{sh} \in \mathbf{G_{sh}}$ *there exists an improving path to any other network* $G'_{sh} \in \mathbf{G_{sh}}$ *and there does not exist an improving path to any network* $G \notin \mathbf{G_{sh}}$ *beyond the basin of attraction.*

Among others, Jackson and Watts (2001)[21] and Hellmann (2013)[20] observed that circuits of improving paths prevent pairwise stable equilibria from existing. Moreover, basins of attraction contain circuits of improving paths.

5. Homogeneous sharing networks

In this section, we assume that all sellers value the good at $v(s_i) = 0$, $i = 1, .., n$ and all buyers value the good at $v(b_i) = 1$, $i = 1, .., m$. This simplifying assumption permits describing the effect of the status quo network on the equilibrium outcome.

5.1. Pairwise stable trades

Corominas-Bosch (2004)[11] shows that any bipartite network G can be decomposed into connected subnetworks of three different types: $G^s(G)$, $G^b(G)$, and $G^e(G)$. We will say that a connected subnetwork of network G belongs to type:

- $G^s(G)$ if it contains more sellers than buyers and any subset of buyers in this subnetwork can be matched with sellers in the same subnetwork;
- $G^b(G)$ if it contains more buyers than sellers and any subset of sellers in this subnetwork can be matched with buyers in the same subnetwork;
- $G^e(G)$ if there is an equal number of sellers and buyers and there is a maximum matching in the subnetwork.

An example of such decomposition is provided in Figure 1.

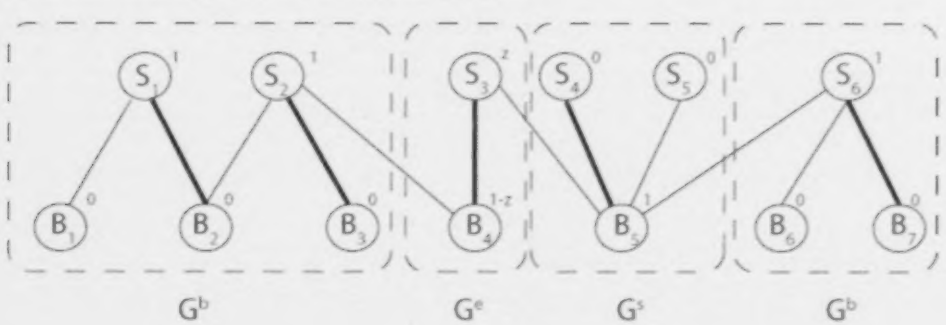

Figure 1: Network decomposition of status quo network G_0.

The decomposition of a network may not be unique, but each seller belongs to a subnetwork of the same type for any decomposition. Furthermore, whenever we refer to a network decomposition, we have in mind the decomposition with the largest number of subnetworks. For any network G, the following properties hold:

- Sellers from $G^c(G)$ can only be connected to buyers from $G^s(G)$ and $G^c(G)$.
- Sellers from $G^s(G)$ can only be connected to buyers from $G^s(G)$.
- Buyers from $G^c(G)$ can only be connected to sellers from $G^b(G)$ and $G^c(G)$.
- Buyers from $G^b(G)$ can only be connected to sellers from $G^b(G)$.

We will use the network decomposition to find the bargaining outcome on a given network. For any network G, we find that the simultaneous bargaining mechanism proposed in this paper leads to the same stable prices and quantities as the sequential mechanism proposed by Corominas-Bosch (2004)[11]:

Theorem 1. *In network G, pairwise stable trades are such that traders of type $G^b(G)$ trade at price of one, traders of type $G^s(G)$ trade at price of zero, and traders of type $G^c(G)$ trade at price of $z \in (0,1)$. If two traders trade the good with each other, they belong to the same subnetwork type.*

This theorem allows us to look at sellers of type $G^s(G)$ as weak sellers, of type $G^c(G)$ as balanced sellers, and of type $G^b(G)$ as strong sellers. In the same way, we call buyers of type $G^s(G)$ strong buyers, of type $G^c(G)$ balanced buyers, and of type $G^b(G)$ weak buyers (see Figure 1 for an example).

Using information sharing, initially weak and initially balanced sellers aim to become stronger. A weak seller $G^s(G_0)$ attempts to increase his trade volumes and prices by getting access from one of the stronger sellers. Balanced sellers $G^c(G_0)$ attempt to increase their prices by getting access from a strong seller. Meanwhile, strong sellers $G^b(G_0)$ aim to share access to their local markets without suffering reductions in prices and volumes. If information exchange leads to a price reduction, such that an initially strong seller $G^b(G_0)$ becomes a balanced seller $G^c(G_{sh})$, this seller requires an adequate compensation for his profit loss. Later in this section, we examine whether there are stable transfers that satisfy the desires of all three types of sellers. The absence of cycles of transfers as specified in Lemma 1 follows directly from the fact that the sequential information sharing eventually leads to a subset of strong sellers $G^b(G_0)$ who do not need to expand their local markets and stop sharing the information with others. This result follows directly from the proofs of Theorems 2 to 5.

5.2. Pairwise stable transfers: networks without weak sellers

In this subsection, we consider status quo networks with initially balanced and initially strong sellers, assuming there are only two subnetwork types: $G^b(G_0)$ and $G^c(G_0)$.

We first solve the game with one strong seller s_1 and many balanced sellers $s_2, ..., s_n$. In this case, the cost of sharing information for s_1 is zero, because sharing does not lead to a decrease in price p_1^*. As a result of information exchange, all sellers become strong $G^b(G_{sh})$. The price of market access $t_{i,j}$ can vary, depending on the equilibrium outcome:

Theorem 2. *In the market with only one strong seller $s_1 \in G^b(G_0)$ and multiple balanced sellers $s_2, s_3, ..., s_n \in G^e(G_0)$, the SPPNS equilibria are such that information sharing increases market prices to $p_i^* = 1$ for any $s_i \in S$, market volume stays at n, and transfers can be of the following types:*

(i) market access is shared free of cost, $t_{i,j} = 0$ for $i, j = 1, ..., n$;

(ii) without loss of generality, s_1 shares his trade connection with s_2 for transfer $t_{2,1} \in (0, 1 - z]$; other expanding sellers get access from s_2 or from each other free of cost.

There is exactly one seller in each subnetwork of type $G^e(G_0)$ expanding the market; the rest of the sellers increase their profits as a result of positive externalities.

This theorem claims that when the cost from sharing information is zero, it is difficult for s_1 to gain much profit from providing market access to other sellers. Moreover, when positive transfers take place, two stories happen. First, seller s_1 discriminates against $n-2$ sellers to keep the information price $t_{2,1}$ positive. Second, seller s_2 shares information freely with other sellers, because any seller s_3 transferring $t_{3,2} > 0$ would get the same information from s_1 at a lower rate.

The following labor market example is a good explanation of how a seller's expansion leads to a greater price.

Example 1. Consider a labor market with two job seekers (s_1, s_2) and three companies (b_1, b_2, b_3) such that s_1 is the only applicant at companies (b_1, b_2) and s_2 is the only applicant at company b_3.

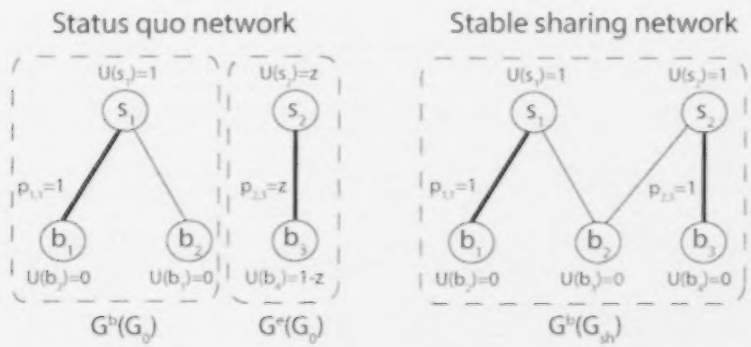

Figure 2: Labor market before and after job seekers exchange information.

The outcome that would result without information exchange is the following: s_1 receives two job offers and accepts any one of them, s_2 receives one job offer and accepts it. We apply Theorem 1 to find the stable wages in the status quo network. Without loss of generality, the corresponding salary is $p_{1,1} = 1$ for "strong" candidate s_1, and $p_{2,3} = z$ for "balanced" candidate s_2. Both workers are equal in their productivity; however, s_1 is offered a higher wage than s_2 because companies b_1 and b_2 compete for having s_1 as an employer.

In order to get a stronger market position, candidate s_2 asks s_1 for the contact information of company b_2. According to Theorem 1, this sharing will increase the demand for candidate s_2 and, therefore, increase

the wage offer for s_2. At the same time, this sharing does not affect job offers of seller s_1, because excess labor demand pushes both prices to maximum. In the equilibrium, any two companies out of three will be matched with the candidates.

This example also provides intuition about the building blocks of labor prices. First, the wage $p_{i,j}$ should exceed the minimum wage $v(s_i)$ required by candidate s_i to become voluntarily employed and stay below the maximum amount $v(b_i)$ that company is willing to pay for the job. Second, the wage is determined by the bargaining power $z \in (0,1)$, which is a measure of worker's negotiation power. Bargaining power z is unconditional on the market structure. Without market competition, parameter z would set the wage at a certain proportion between worker's cost of labor and company's value of labor. Third, the wage of a certain worker depends on local market competition. Information frictions on the labor market make local competition more important than global competition, even if the market represents a one-component graph.

This example fits well into the literature on labor networks. As Grannovetter (1973)[18] finds, workers often find jobs through information exchange with other workers. His findings are consistent with the intuition provided in this example: workers from different subnetworks can exchange information about the job openings freely if their own job position is secured.[6]

The result of Theorem 2 can be extended to the cases where there are multiple sellers of types $G^b(G_0)$ and $G^c(G_0)$. If there are multiple initially strong sellers, it is straightforward and given without formal proof that market access is always free, because the cost of sharing is zero and the information market becomes competitive:

Theorem 3. *In a market with multiple strong sellers $G^b(G_0)$ and balanced sellers $G^s(G_0)$, SPPNS equilibria are such that information sharing increases market prices to $p_i^* = 1$ for any $s_i \in S$, market volume stays at n, and market access is shared free of cost.*

This theorem claims that if all sellers are already profitable in G_0, they will freely share market access to increase market prices to maximum. In the next section, we check if these results hold when non-trading sellers are present at G_0.

Additionally, in Theorem 14 shown in Appendix E, we prove that when default rules are not necessarily pro rata, the information can be sold sequentially, such that multiple sellers pay positive transfers for market access. These equilibria exist for $s_1 \in G^b(G_0)$ and $s_2, ..., s_n \in G^c(G_0)$ only when buyers have more bargaining power than sellers, $z \leq \frac{1}{2}$, and the default rules are such that few balanced sellers $s_i \in G^c(G_0)$ pay full transfer $t_{i,j}$ only if they stay solvent ($F_{i,j} \leq t_{i,j}$). However, when there are multiple strong sellers $G^b(G_0)$, the information is only shared free of cost, as in Theorem 3. Some intuition about this outcome is also given in Appendix E.

[6] See, for example, Calvo-Armengol (2004)[9] for a more detailed consideration for the labor market networks.

5.3. Pairwise stable transfers: networks with weak sellers

In this subsection, we consider status quo network G_0, which contains a few subnetworks of each type: $G^s(G_0)$, $G^c(G_0)$, and $G^b(G_0)$.

Theorem 4 extends the result of Theorem 3 to all markets with excess market demand.

Theorem 4. *In a market with more buyers than sellers, $m > n$, the SPPNS equilibria are such that information sharing increases market prices to $p_i^* = 1$ for all $s_i \in S$, market volume increases to n, and market access is shared free of cost.*

In Theorem 4, we show that matching frictions created by any network structure G_0 can be overcome when sellers exchange market information. However, the information exchange completely reallocates trade surplus from buyers to sellers. The following example provides intuition about Theorem 4.

Example 2. Consider a market of farmers and consumers. Different locations have different crops because of seasonal and geographical specifics. According to Theorem 4, if there is excess national demand, each farmer is able to sell his crop either in the local marketplace or in another location with the help of intermediaries, independent of the initial geographical location. Moreover, if the local marketplace initially has an excess supply, the price in this location will still increase to maximum and the excess crop will be exported to other locations. Thus, information sharing among sellers minimizes the effect of local competition on local prices.

The effect of information sharing on prices is less certain if there are more sellers than buyers ($n \geq m$). In this case, market sharing may create negative externalities. In the game with pairwise agreements, negative externalities may lead to the non-existence of pairwise stable transfers. When SPPNS equilibrium does not exist, we use the concept of basins of attraction. The following example helps to reveal the nature of negative externalities.

Example 3. Consider a network with two strong and one weak sellers: s_1,$s_2 \in G^b(G_0)$ and $s_3 \in G^s(G_0)$ as presented in Figure 3.

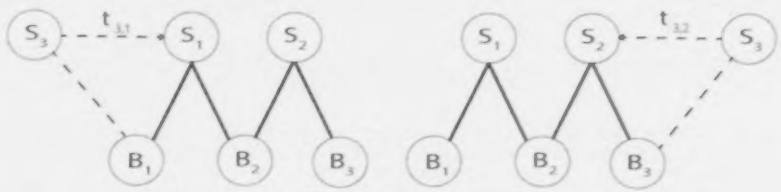

Figure 3: Example with two strong and one weak sellers when pairwise stable transfers do not exist.

Assume sellers have at least twice as much bargaining power than buyers: $z > \frac{2}{3}$. Under this assumption, market sharing would increase the total profit $\sum_{i,j}(p_{i,j} - v(s_i)) = 2$ of all sellers to $3z > 2$. However, SPPNS equilibrium does not exist in this game because of the negative externalities that two sharing sellers impose on the one non-sharing seller. To see this, suppose that s_1 shares access with s_3 in exchange for $t_{3,1} \in [1-z, z]$

such that both s_1 and s_3 do not have incentives to breach the contract. The information sharing changes sellers' payoffs from

$$U(s_1) = 1,\ U(s_2) = 1,\ U(s_3) = 0$$

to

$$U(s_1) = z + t_{3,1} \geq 1,\ U(s_2) = z,\ U(s_3) = z - t_{3,1} \geq 0.$$

However, these strategies are not pairwise Nash stable because, among other deviations, s_2 and s_3 want to deviate to transfer $\hat{t}_{3,2} \in (0, 1-z)$ and breach the sharing agreement between s_3 and s_1. In the case of deviation, the profits change in favor of s_2 and s_3:

$$U(s_1) = z,\ U(s_2) = z + \hat{t}_{3,2} < 1,\ U(s_3) = z - \hat{t}_{3,2} > 0.$$

The last strategies are not stable either because seller s_2 will prefer to breach contract with s_3 and increase his utility back to one.

The three types of deviations presented above form a circuit that prevents pairwise stable equilibrium from existing. As intuited from the example, the main reason for instability is that the property rights on information are not assigned. In Section 7, we find stable equilibria given the assumption that the information property rights are transferred when the information is sold. This alternative setup helps to reduce multiplicity in some cases. In this section, we solve the problem of equilibrium existence by providing an alternative notion of stability.

In markets with more sellers than buyers, the effect of information sharing on prices and trades depends on the bargaining power z of sellers relative to buyers. We first show that when sellers have less bargaining power than buyers, information sharing will increase the total trade volume but still reallocate trade surplus to the side of sellers.

Theorem 5. *In the market with more sellers than buyers, $m < n$, if buyers have more bargaining power, $z < \frac{1}{2}$, and the basin of attraction is such that information sharing increases prices to $p_i^s = 1$ for all initially strong and balanced sellers $G^c(G_0) \cup G^b(G_0)$ and some initially weak sellers $G^s(G_0)$, and keeps the price at $p_i^s = 0$ for the remaining weak sellers. Market volume increases to $m - 1$.*

The basin of attraction is such that, in each network, sellers from different subnetworks $G^b(G_0)$ and $G^c(G_0)$ combine into one subnetwork $G^b(G_{sh})$ and share all non-trading buyers but one with sellers from $G^s(G_0)$.

It is crucial that buyers have more bargaining power than sellers for the information sharing to be limited. This is because the bargaining power z determines sellers' profit from matching two non-trading players. Therefore, when z is small, sellers are not willing to sacrifice their monopoly rents to match non-trading players. As a result, sellers match new traders only if it does not affect their competitiveness in the market.

In some cases, information sharing can be beneficial for buyers—when non-trading buyers get connected to new sellers and the prices drop because of increased competition among sellers. The next theorem shows that when the bargaining power of sellers increases above $z = \frac{2}{3}$, strong sellers may become balanced sellers by sharing their market access with weak sellers. This happens because the profit from matching non-trading players covers the loss from dissolved monopoly rents.

Theorem 6. *In the market with more sellers than buyers, $m < n$, if sellers have twice more bargaining power, $z > \frac{2}{3}$, and each weak buyer $G^b(G_0)$ is connected to only one seller in G_0, the SPPNS equilibria are such that information sharing changes prices to $p^* = z$, market volume increases to m, and strong sellers $G^b(G_0)$ share market access with weak sellers $G^s(G_0)$ at $t = z$.*

In the markets that satisfy properties of Theorem 6, sharing does not create negative externalities. When sharing creates negative externalities and $z > \frac{1}{2}$, the total volume would still increase above the initial market volume but the prices would not necessarily drop to z. Example 3 demonstrates this multiplicity of prices.

5.4. Market efficiency and Walrasian outcome

We compare the equilibrium prices and quantities to the Walrasian middle price and quantities corresponding to the intersection of supply and demand curves. In the homogeneous case, Walrasian market volume is $Q^{walr} = \min(m, n)$ and price is $p^{walr} = \frac{1}{2}$. According to Theorems 4 to 6, market sharing often increases trade volume but not necessarily to the Walrasian level.

Corollary 1. *In order for information sharing to increase total market trade to the Walrasian level, it is necessary that either there be more buyers than sellers, $m > n$, or that sellers have more bargaining power, $z > \frac{1}{2}$.*

The next corollary shows that buyers who did trade a priori may benefit from information sharing only when the market is dominated by sellers with high bargaining power. Otherwise, information sharing leads to a complete reallocation of market surplus to sellers.

Corollary 2. *Buyers can benefit from information sharing only in the markets with more sellers than buyers, $m < n$, and with sellers having more bargaining power than buyers, $z > \frac{1}{2}$.*

It is noteworthy that the results of Corollary 1 and Corollary 2 hold for any status quo network G_0. Bargaining power z becomes the driving factor in the model. It is helpful to understand the meaning of this parameter using an example.

Example 4. Consider a market for used automobiles and a market for new automobiles. If we think of sellers' bargaining power z as measure of haggling, we predict that in the market for used automobiles, information sharing is likely to be more beneficial for buyers than in the market for new automobiles. Prices for new automobiles are usually publicly available on the Internet, while the quality of used automobiles is often unknown. We can interpret this as if the bargaining power of sellers relative to buyers z is greater in

the market for used cars. Therefore, if the difference between the bargaining powers is significant, sellers have more incentives to drive used automobiles rather than new automobiles from other locations in order to sell them locally. According to the model, if non-trading buyers do not travel to other locations themselves, and sellers have at least twice more bargaining power, market sharing leads to a greater number of sales and lower prices for these buyers. It is also true that not all sellers will expand their local markets. Market expansion may impose both positive and negative externalities on other sellers. Therefore, only a limited number of sellers will gain or provide market access. While this example does not account for many specifics of the automobile market, such as heterogeneity of brands and car specifics, it provides some intuition behind the mechanism of exchange that is common among car dealers.

6. Heterogeneous sharing networks

In this section, we relax the assumption that all buyers (sellers) value goods equally among themselves. To avoid problems with ties, we assume that all traders have different values.

The stable price of any trade is determined based on the alternative options. According to Assumption 3, price $p_{i,j}$ is chosen in proportions z and $1-z$ in the interval $[\underline{p_{i,j}}, \overline{p_{i,j}}]$. We now show that the price interval is determined by outside offer $\underline{p_{i,j}}$ of seller s_i and outside offer $\overline{p_{i,j}}$ of buyer b_j.

Definition 4. An outside offer is a stable price at which a trader would trade if its current trade counterparty was removed from the network.

It becomes clear that sellers want to access buyers with higher values to increase their outside options and, as a result, price and quantity. One new connection is sufficient to increase the price of a trading seller and two new connections are sufficient to increase the price and volume of a non-trading seller.[7] Therefore, the market expansion process considered in this paper becomes equivalent to the process of finding trade counterparties and counterparties for outside offers.

6.1. Passive and active traders

To solve for the equilibrium, we propose to separate traders into three categories in a given network G according to the stable trade and outside offers. By definition, the node decomposition is unique for each network G.

- **Active traders** $N_A(G)$ is a set of buyers and sellers who trade the good in a pairwise stable matching in G.
- **Passive traders** $N_P(G)$ is a set of non-trading buyers and sellers that provide outside offers for $N_A(G)$.
- $N_N(G)$ is a set of non-trading buyers and sellers that do not provide outside offers to $N_A(G)$.

[7] This intuition can be used to show that transfers do not form cycles. As a result, the defaults are uniquely determined.

Using this decomposition, we find that, as a result of information sharing, buyers with the lowest values do not trade and do not affect market prices of those who trade.

Theorem 7. *In SPPNS equilibria, any passive or active buyer $N_P(G_{sh})$ values goods higher than any non-trading buyer $N_N(G_{sh})$. In the basins of attraction, there is at least one network G_{sh} where the same is true.*

Theorem 7 claims that given k stable trades, the buyers who trade belong to the set of $2k$ buyers with the highest values. However, it is not the case that the trading buyers value goods the most. Sellers may intentionally leave high valued buyers as passive to increase their market power. However, as Theorem 8 shows, when sellers have more bargaining power than buyers, all trading buyers value goods higher than all non-trading buyers.

Theorem 8. *If sellers have more bargaining power than buyers, $z > \frac{1}{2}$, the following property holds in SPPNS equilibria: any active buyer $N_A(G_{sh})$ values good higher than any passive or non-trading buyer $N_P(G_{sh}) \cup N_P(G_{sh})$. In each basin of attraction, there is at least one network G_{sh} where the same is true.*

This theorem answers one of the main questions raised in this paper: when and how the information exchange among sellers may benefit buyers. We proved that the pairwise information exchange among sellers and competitive prices are sufficient conditions to allocate goods to buyers with the highest values if sellers are guaranteed a sufficient surplus from information exchange.

6.2. Market efficiency and Walrasian outcome

In this paper, market efficiency is considered in terms of the total market surplus:

$$\sum_{i=1}^{n}\sum_{j=1}^{m}(v(b_j) - v(s_i))q_{i,j}.$$

We denote Walrasian market volume as Q^{walr} and the midpoint Walrasian price as v^{walr} (see Figure 4). The maximum trade surplus is generated when sellers with the lowest production costs $s_i : v(s_i) \leq v^{walr}$ trade with the buyers with the highest values $b_j : v(b_j) \geq v^{walr}$.

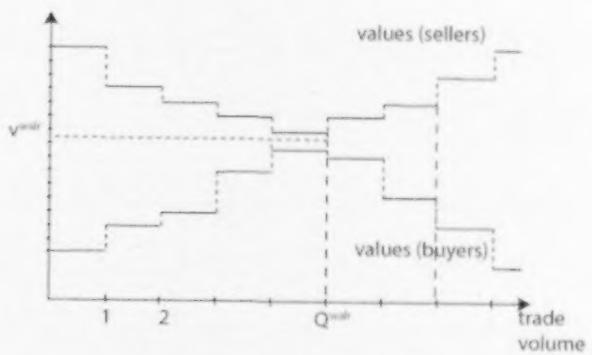

Figure 4: Walrasian trade volume and price.

We first show that Walrasian market volume and trade surplus can be generated in the complete network.

Lemma 2. *In the complete network G_{sh}, stable trades generate Walrasian market volume and Walrasian total trade surplus.*

This Lemma is given without a proof and follows directly from the fact that stable assignments form a complete lattice. It becomes sufficient to show that, with all prices being equal to $p = p^{walr}$, a matching that connects all sellers with values $v(s_i) \leq v^{walr}$ to all buyers with values $v(b_j) \geq v^{walr}$ in any arbitrary order is stable. This fact is straightforward because trading sellers and buyers do not have incentives to deviate to alternative trades with each other or with non-trading players. Any deviation would lead to lower prices for sellers and higher prices for buyers.

Following the same logic, we can impose a weaker condition on the network to generate the maximum trade surplus.

Lemma 3. *If there is a maximum matching between sellers $\{s_i : v(s_i) \leq v^{walr}\}$ and buyers $\{b_j : v(b_j) \geq v^{walr}\}$, the stable trades generate Walrasian market volume and Walrasian total trade surplus.*

In the homogeneous networks, considered in Section 5, maximum trade surplus corresponded to the maximum trade volume. In the heterogeneous networks, an increase in the trade volume may reduce the total market surplus. It happens when an increase in market volume leads to higher prices for previously traded buyers and lower prices for previously traded sellers. Moreover, if the market is inhabited with sellers who have values $v(s_i) > v^{walr}$ and buyers who have values $v(b_j) < v^{walr}$, Walrasian allocation does not guarantee the maximum trade volume.

In Section 5, we showed that, in stable networks, total trade volume may stay below Walrasian outcome, if sellers prefer to keep non-trading buyers to increase competition and current market prices. Example 5 shows that, in the heterogeneous case, total trade volume may also be above the Walrasian trade level.

Example 5. Consider an example with two countries: rich country with seller s_1 and buyer b_1, and poor country with seller s_2 and buyer b_2. The production costs and consumption values are indicated in Figure 5. To consider different cases, we keep the consumption value of buyer b_2 as parameter $x \in (1, 6)$.

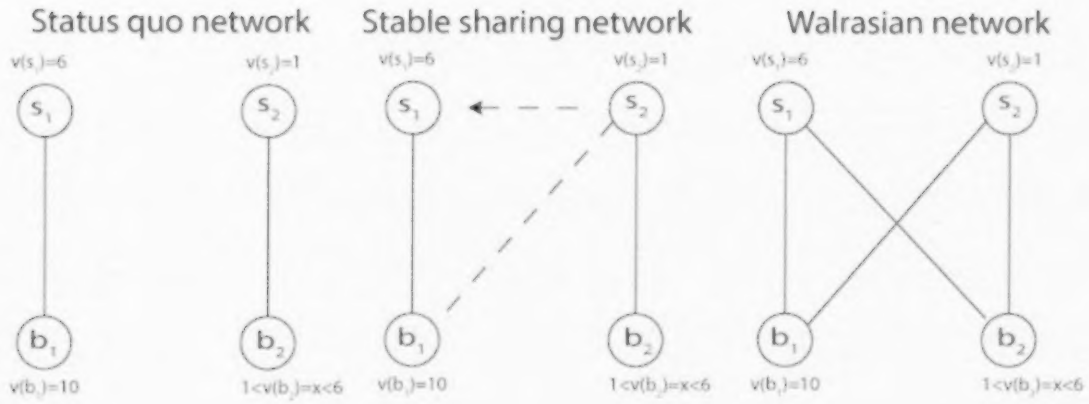

Figure 5: Market structure of Example 5

a) When each seller trades in the domestic market, the price of goods in the rich country is

$$p_{1,1} = 10z + 6(1-z) = 6 + 4z,$$

and in the poor country, it is

$$p_{2,2} = xz + 1(1-z) = 1 + (x-1)z.$$

Global trade volume is two units and global trade surplus is

$$\sum_{i=1}^{n}\sum_{j=1}^{m}(v(b_j) - v(s_i))q_{i,j} = 3 + x.$$

The surplus that sellers make in both countries is fraction z of the total trade surplus

$$\sum_{i=1}^{n}\sum_{j=1}^{m}(p_{i,j} - v(s_i))q_{i,j} = (3+x)z.$$

b) In the Walrasian outcome, poor seller s_2 trades with rich buyer b_1, while traders s_1 and b_2 do not trade (see Figure 6). Because we assumed limited production capacity, sales in the poor country shut down.

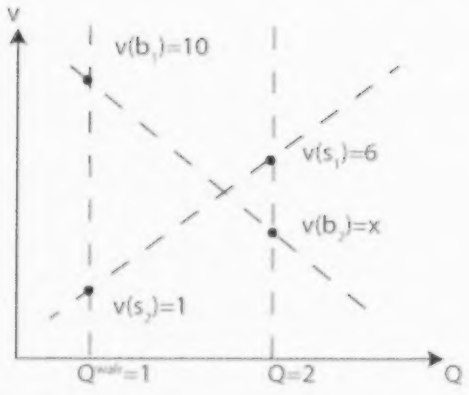

Figure 6: Supply and demand schedule for Example 5.

c) Consider the case where seller s_1 serves as an intermediary for seller s_2. Therefore, rich seller s_1 earns profit not from production itself but from channeling the good from poor country sellers to rich country buyers.

The price at which seller s_2 trades with buyer b_1 is

$$p_{2,1} = 6z + x(1-z) = x + (6-x)z.$$

Global trade volume is one unit and the sellers gain surplus

$$\sum_{i,j}(p_{i,j} - v(s_i))q_{i,j} = x - 1 + (6-x)z.$$

Stable transfer $t_{2,1}$ exists when the joint surplus of sellers increases, which is true when

$$3z - 1 \geq (2z-1)x.$$

The relationship between the bargaining power z and the equilibrium network structure G_{sh} is not straightforward. If buyers have more bargaining power, $z \leq \frac{1}{2}$, countries prefer to trade in their local markets rather than to share markets. In this case, global trade volume is above the Walrasian level, and total trade surplus is below the Walrasian level. If sellers have more bargaining power, $z > \frac{1}{2}$, the equilibrium network may be either the Walrasian network or the status quo network G_0, depending on value x. When poor buyer b_2 is willing to pay a large amount x for the good, poor seller s_2 sells the good to the poor buyer b_2. When x is small, poor seller exports the good to rich buyer b_1.

This example is also illustrative because it shows that, in heterogeneous networks, the interaction among sellers sharing information may lead to a decrease in the total trade volume, while in the homogeneous case, information sharing always leads to an increase in the total trade volume.

As Example 5 showed, the effect of information sharing on the market prices and quantities is not obvious. For a general class of networks, we would like to know whether there is a set of parameters where information

sharing among sellers makes market volume and total trade surplus equal or close to the Walrasian outcome. The following theorem sheds light on this issue:

Theorem 9. *If sellers have more bargaining power than buyers, $z > \frac{1}{2}$, the SPPNS equilibria are such that market trade volume weakly exceeds Q^{walr}, and the set of trading buyers includes the set of buyers who would trade in the Walrasian outcome.*

In each basin of attraction, there is at least one network G_{sh} where the same is true.

As Theorem 9 claims, in the markets with powerful sellers, a deviation from the Walrasian outcome can only be due to overproduction. The overproduction happens when the most productive sellers cannot reach buyers with the highest values. As a result, the most productive sellers are underpaid and the most productive buyers overpay. If sharing is insufficient to reach the maximum trade surplus, the positive effect is that a greater number of traders can sell or buy the good compared with the number that could do so in the Walrasian outcome.

7. Information sharing with transferable rights

In this section, we consider information sharing with transferable rights as an alternative to the non-transferable rights considered in Section 6. It means that a seller takes responsibility to cease trade with the buyers once he reveals the information about these buyers to other sellers. In some markets, this form of communication may be considered a collusion of sellers and be illegal. In other markets, it is well accepted that one seller delegates sales to other sellers. Franchising or overseas expansion can be considered a process of market sharing with transferable property rights. We will use homogeneous networks to show the importance of information sharing rules on the equilibrium outcome.

The main difference between information sharing with transferable rights and non-transferable rights is the cost of sharing information. The first case that we consider is a good illustration of why transferable property rights are a barrier for extensive information sharing.

We consider a homogeneous status quo network with initially balanced and initially strong sellers, assuming there are only two subnetwork types: $G^b(G_0)$ and $G^e(G_0)$. We denote the number of subnetworks of type $G^b(G_0)$ as $|G^b(G_0)|$.

Theorem 10. *In the market with one or multiple strong sellers $G^b(G_0)$ and balanced sellers $G^s(G_0)$, SPPNS equilibria are such that information sharing with transferable rights leaves the prices of all initially strong sellers at $p^s = 1$; and changes the prices of at least $m - n - |G^b(G_0)|)$ subnetworks $G^e(G_0)$ to $p^s = 1$, the rest of prices stay at $p^s = z$. Market volume stays at n.*

Prices upgrade because each subnetwork with strong sellers $G^b(G_0)$ leaves one non-trading buyer in the subnetwork and shares access to the rest of non-trading buyers with sellers $G^e(G_0)$.

According to the theorem above, information sharing with transferable property rights leads to prices equal to or lower than information sharing without property rights. This result could be because information

sharing with transferable rights limits positive externalities that it creates for non-sharing sellers in the form of higher market prices. When non-trading buyers are isolated, average sellers face less competition and cannot raise their prices to maximum.

We can extend this result for the status quo network G_0 that contains a few subnetworks of each type: $G^s(G_0)$, $G^c(G_0)$, and $G^b(G_0)$.

Theorem 11. *In the market with more buyers than sellers, $m > n$, the SPPNS equilibria are such that information sharing with transferable rights leads to lower prices and market volumes than information sharing with non-transferable rights.*

Because the stable outcome described in Theorem 4 leads to the maximum trade volume and maximum prices, in order to prove Theorem 11, we just need to provide two counterexamples, where not all sellers trade and where the price of at least one pair of traders is less than one.

Example 6. Consider a network G_0 presented in Figure 7. In the status quo, two trades take place at prices $p_{1,2} = 1$ and $p_{2,3} = z$. If sellers share information without restrictions on their individual trades (in non-transferable fashion), information sharing leads to three trades with the prices of ones. Alternatively, information sharing with transferable rights leads to the stable outcome of either prices or quantities being less than the ones with non-transferable information rights. Moreover, the example shows that information sharing with transferable rights leads to a greater segregation among local markets.

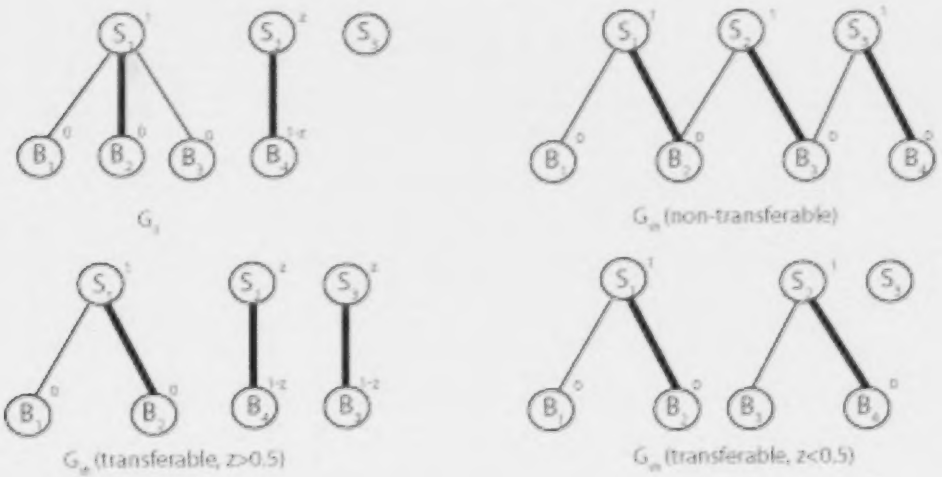

Figure 7: Effect of transferable information rights on stable market prices and volumes.

In the markets dominated by sellers, we also observe that information sharing with transferable rights is less aggressive than with non-transferable rights. Information sharing with transferable rights does not change the number of connections in the network and does not reduce segregation among local markets. The next theorem provides an alternative to Theorem 5 for the case of transferable rights.

Theorem 12. *In the market with more sellers than buyers, $m < n$, if buyers have more bargaining power, $z < \frac{1}{2}$, the basin of attraction is such that information sharing with transferable rights leaves the prices of all initially strong sellers at $p^s = 1$, changes the prices of at least $m - n - |G^b(G_0)|)$ balanced and weak sellers to $p^s = 1$, and the price of at most one balanced seller to $p^s = z$. The rest of prices stay at $p^s = 0$. Market volume increases to less than $m - |G^b(G_0)|$.*

Prices upgrade because each subnetwork with initially strong sellers leaves one non-trading buyer in the subnetwork and shares access to the rest of non-trading buyers with initially balanced and weak sellers, preferring balanced sellers.

The next theorem shows that in the highly segregated markets dominated by sellers, information sharing leads to the same outcome independent of the property rights rules.

Theorem 13. *In the market with more sellers than buyers, $m < n$, if sellers have twice more bargaining power, $z > \frac{2}{3}$, and each weak buyer $G^b(G_0)$ is connected to only one seller in G_0, information sharing with transferable rights leads to SPPNS equilibria with the same total volume of trade as information sharing with non-transferable rights described in Theorem 6.*

This theorem describes a special case of the extremely segregated market where sellers do not impose negative externalities on each other and where information sharing is beneficial for the sellers because of their high bargaining power z. Therefore, both regimes of information sharing lead to the same outcomes.

8. Conclusion

The main purpose of this paper is to analyze how information sharing among sellers may eliminate barriers created by the networked market structure. Information exchange among sellers is modeled as a process of sharing access to potential buyers. The question that we address is how the market expansion resulting from information sharing affects prices, trade quantities, and trade surplus.

We show why it is important to account for asymmetric industry formation, when only one side of the market can expand market access. In the field of microeconomics, the structure of the market is often assumed to be given: monopoly, oligopoly, perfectly competitive market. In this paper, we consider a market with a network structure, where the monopoly power of each seller is determined locally. Each local market may favor either sellers or buyers depending on the number of traders on each side and on the traders' values. We show that when sellers interact with each other, they can completely change the market structure. As a result, prices and volumes will also change because traders will make their trading choices conditional on the updated trade offers.

We first research a homogeneous version of the model, assuming that all sellers produce at the same costs and buyers value goods equally. In this case, sellers share market access to achieve market expansion and not cost reduction. We show that information sharing always increases total trade volume and trade surplus. In order for the market trade volume to increase to maximum, sellers must have enough incentives

to share information. This happens when there are more buyers than sellers or when sellers have more bargaining power relative to buyers. For buyers to benefit from the sellers' interactions, it is necessary to have more sellers than buyers, with sellers having more bargaining power than buyers have. Only in this case, information sharing may lead to a decrease in market prices and increase in the buyers' trade surplus.

One special case of the homogeneous networks is where all sellers initially operate at full capacity and share information to promote themselves and increase prices. In addition to the equilibrium where the firm earns zero profit from sharing, another, non-trivial, equilibrium exists, where information is shared at a positive price with only a limited number of sellers. This result shows that when information sharing does not increase the volume of trade, it is difficult to keep the price of the information positive and to resell the information multiple times.

We show in the paper that, in the general case, sharing may lead to either an increase or decrease in the total trade volume. However, when sellers have more bargaining power than buyers, the total trade volume will be at least at the Walrasian level. Moreover, the buyers who value goods the most would be guaranteed to trade. It means that market inefficiency can only be due to overproduction, which happens because firms with high production costs strategically do not share information with low production firms. This result makes sense when applied to the market of geographically separated companies. In particular, we explain why high-cost sellers may continue producing for their local markets instead of importing goods from other locations if the production capacities of the sellers are limited. This outcome may happen when the selles' profit from each trade is sufficiently high and, as a result, sellers as a group prefer to increase their volume of trade rather than decrease production cost.

The paper also shows that, as a result of information sharing, the buyers who value goods the least will be squeezed out from the market independent of the their location and bargaining power, while the same is not the case for sellers. In the example with geographically separated companies, this result is possible but does not always hold because of transaction costs, tariffs, and entry costs. For instance, the model does not account for the cost of information exchange that could reduce incentives of buyers to share market access. The model can be improved to account for these additional costs. Moreover, it would be interesting to know how information sharing affects markets with flexible production capacities.

In the last section, we provide alternative information sharing rules. In particular, we assume that information is transferred together with property rights—a seller needs to abstain from trade with a buyer when he shares the information about this buyer, thus completely redirecting business with this buyer to another seller. Using the homogeneous network setup, we show that information sharing with property rights leads to the outcomes with lower market volume and lower prices compared with information sharing without property rights. This result makes us think about how the information exchange should be regulated not only to benefit sellers, but also to protect buyers. Even though imposing property rights on information exchange will prevent prices from reaching extremes, information exchange will generate lower trade volumes. Therefore, in the future, it would be interesting to consider a trade-off between breaking the network barriers

and keeping the market prices reasonable.

The next step that should be undertaken is to understand how policy makers should reallocate the market surplus in cases where information exchange among sellers makes local markets sufficiently monopolistic and significantly reallocates market surplus to the sellers' side. Further extensions of the model may include relaxing the key assumptions, such as linearity of trader's preferences and homogeneity of good.

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Appendixes

Appendix A: Proof of Lemma 1

To find default indicators, it is necessary and sufficient to find a fixed point $D = (D_1, .., D_n)$ of a vector-valued function $\Phi = (\Phi_1, ..., \Phi_n)$ defined on the complete lattice $\{0, 1\}^n$, such that $D_j = \Phi_j(D)$. *Ceteris paribus*, an increase from $D_i = 0$ to $D_i = 1$ in equations of type (1) leads to either a change from $\Phi_j(D) = 0$ to $\Phi_j(D) = 1$ or no change. That is, Φ is an order-preserving function. This observation allows us to use Tarski fixed-point theorem to prove that the set of fixed points of Φ on $\{0, 1\}^n$ is a complete lattice itself.

Consider a set of solutions with minimum element $D^{Min} = (D_1^{Min}, .., D_n^{Min})$ and maximum element $D^{Max} = (D_1^{Max}, .., D_n^{Max})$. According to the definition of a complete lattice, there are two sets of sellers: one set with the default status being the same for all fixed points and another set of sellers with the default indicators being one in D^{Max} and zero in D^{Min}. Without loss of generality, we assume that

$$D_i^{Min} = D_i^{Max} \text{ for all i}\leq\text{x},$$

$$D_i^{Min} = 0,\ D_i^{Max} = 1 \text{ for all i}>\text{x}.$$

Consider seller s_i with $i > x$. Comparison of equation (1) for D^{Min} and D^{Max} makes it clear that there is at least one more seller s_{j_1}, $j_1 > x$, causing the default of seller s_i. If no such seller exists, expressions on the right hand side of equation (1) should be identical for D_i^{Min} and D_i^{Max}, but we selected i such that $D_i^{Min} \neq D_i^{Max}$. Following the same logic, we say that there is at least one more seller j_2, causing the default of j_1. After repeating the same step at most N times, we reach the point when we return to one of the sellers $(i, j_1, j_2, ..., j_{k-1})$. Therefore, we showed that in order for $D^{Min} \neq D^{Max}$ there should be a cycle in the subnetwork of positive transfers among sellers. Consequently, we proved that in the acyclic networks, the default indicators are uniquely determined. ►

Appendix B: Proof of Theorem 1

First, we prove that all sellers in $G^b(G) \cup G^e(G)$ and all buyers in $G^s(G) \cup G^e(G)$ belong to a stable assignment. We only need to prove this for seller $s_i \in G^b(G) \cup G^e(G)$. The result for buyers $b_i \in G^s(G) \cup G^e(G)$ follows by symmetry.

Suppose to the contrary, s_i does not trade. Any buyer b_j connected to s_i does not have incentive to trade with s_i only when b_j trades at price $p_{kj} = 0$ with some other seller s_k. Therefore, seller s_k also gets zero payoff $v(s_k) - p_{kj} = 0$. According to the properties of network decomposition, if $b_j \in G^b(G) \cup G^e(G)$, it can only be connected to $s_k \in G^b(G) \cup G^e(G)$. We return to the same initial condition as with seller s_i: in the equilibrium, s_k gets a payoff of zero and does not have profitable deviations. After repeating this sequence of logic statements multiple times, we find that for a set of connected subnetworks of type $G^b(G) \cup G^e(G)$, all buyers trade at zero prices, and at least one seller s_i does not trade at all. Given that there are more buyers than sellers in $G^b(G) \cup G^e(G)$, we reach a contradiction: the number of buyers trading with counterparties in the subnetworks $G^b(G) \cup G^e(G)$ is greater than the number of sellers in the same subnetworks. This proves that any seller s_i in $G^b(G) \cup G^e(G)$ belongs to a stable assignment. By symmetry, all buyers in $G^s(G) \cup G^e(G)$ belong to a stable assignment.

To find the payoffs of sellers in $G^b(G)$, consider the largest union of subnetworks $G^b(G)$ connected. In the equilibrium, there is at least one buyer b_j who is unmatched. For the pairwise deviation of two connected traders (s_i, b_j) not to exist we require $p_{ik} = 1$ for some b_k. For the same reason, all buyers connected to s_i gain payoffs of zero, and all sellers that share at least one buyer with s_i trade at the price of one. Since each seller s_i is connected to at least two buyers and each buyer is connected to a seller, this property is extended to all sellers in the subnetworks $G^b(G)$. It proves that sellers in $G^b(G)$ trade at price of one. In the same way it can be proved that all buyers in $G^s(G)$ trade at the price of zero.

Next, we prove that in a subnetwork of type $G^e(G)$ all traders belong to a maximum assignment. According to the properties of network decomposition, there is a maximum assignment in $G^e(G)$, such that all sellers and all buyers trade. If there is at least one maximum assignment that is stable, all stable assignments will be maximum. This follows from the fact that the set of stable assignments coincides with the core (Shapley and Shubik (1971)). To see why the core is not empty, consider a stable matching with all pairs in $G^e(G)$ trading at the same price. Under these contracts, deviations within the subnetwork are not possible. In particular, sellers of $G^e(G)$ also do not have incentives to deviate to trades with buyers from $G^s(G)$, and buyers of $G^e(G)$ do not have incentives to deviate to trades with sellers from $G^b(G)$.

Given that sellers of type $G^b(G)$ trade at the price of one and buyers of type $G^s(G)$ trade at the price of zero, the set of stable outcomes in a subnetwork $G^e(G)$ forms lattice with prices between zero and one. Therefore, the minimum utility for each trader $s_i, b_j \in G^e(G)$ is $\underline{u}(s_i) = \underline{u}(b_j) = 0$. Given Assumption 3, the equilibrium payoffs for $s_i, b_j \in G^e(G)$ are given according to $p_{i,j} = z$:

$$u(s_i) = p_{i,j} - v(s_i) = z,$$

$$u(b_j) = v(b_j) - p_{i,j} = 1 - z.$$

►

Appendix C: Proof of Theorem 2

We first notice that $s_1 \in G^b(G_{sh})$, because sharing with a seller from $G^c(G_{sh})$ does not decrease the initial price of s_1. For the same reason, all sellers of type $G^c(G_0)$ turn into type $G^b(G_{sh})$. Besides, there is exactly one seller in each subnetwork of type $G^c(G_0)$ paying for access, while the rest of the sellers free ride, because an expansion of one seller is enough to change the whole subnetwork from $G^c(G_0)$ to $G^b(G_{sh})$.

Strategies with zero transfers constitute an equilibrium because sellers $(s_2, ..., s_n)$ do not have incentive to deviate to an outcome with lower profits.

To find equilibria with non-zero transfers, we fix tranfer $t_{2,1} \geq 0$ from s_2 to s_1 and explore stable transfers of another seller s_3 to either s_1, or s_2, or both. First, suppose that both sellers s_2 and s_3 buy access directly from s_1 and that at least one of them pays a positive transfer (e.g. $t_{3,1} > 0$). Then, s_3 has incentive to deviate and get access from $s_2 \in G^b(G_{sh})$ at a lower transfer while s_2 has incentive to provide the access in exchange for $t_{3,2} > 0$. To avoid a deviation of this kind, we consider the case where s_1 and s_3 are not sharing and where s_3 buys access from s_2 at $t_{3,2} \geq 0$. If the transfer is strictly positive, $t_{3,2} > 0$, sellers s_1 and s_3 will deviate to a lower transfer $\tilde{t}_{3,1} = t_{3,1} - \varepsilon$, which will benefit seller s_3 and also seller s_1 because

$$\tilde{t}_{3,2} + \min(1, \Delta t_{2,1} + t_{3,2}) > t_{3,2} + \Delta t_{2,1}.$$

We have proved that a blocking pairwise deviation exists when transfer $t_{3,2}$ is positive. To avoid this kind of deviations, we require that $t_{3,2} = 0$.

The next observation that we make is that a net transfer that s_2 pays to s_1 should not exceed the value that the information sharing adds to a coalition, meaning that $\Delta t_{2,1} \leq 1 - z$. It follows that, in the equilibrium, $\Delta t_{2,1} = t_{2,1} \in [0, 1 - z]$.

Finally, because s_3 gets access at zero cost, we require all remaining transfers to be zero to prevent other sellers from deviating together with s_2 or s_1.►

Appendix D: Alternative default rules

The following theorem shows that when any two sellers s_i and s_j can strategically choose default transfer $F_{i,j}$, more sellers can pay non-zero transfer for the information. We change the cooperative game by expanding the players' action space to show how sensitive the macro properties of the market are to the contracting rules.

Theorem 14. *In the market with only one strong seller $s_1 \in G^b(G_0)$ and multiple balanced sellers $s_2, s_3, ..., s_n \in G^c(G_0)$, the SPPNS equilibria are such that information sharing increases market prices to $p_i^S = 1$ for any $s_i \in S$, market volume stays at n, and transfers can be of the following types:*

(1) market access is shared free of cost, $t_{i,j} = 0$ for $i, j = 1, ..., n$;

(2) the information can be transferred sequentially from seller s_1 to a subset of intermediaries $(s_{(2)}, s_{(3)}, ..., s_{(N^{int})})$, such that each intermediary $s_{(i)}$ transfers amount $t_{(i+1),(i)}$ that was transferred to him and adds a positive net transfer $\Delta t_{(i),(i-1)}$, such that

$$\Delta t_{(i),(i-1)} \in [F_{(i),(i-1)},\ 1-z]$$

for all $i \leq N^{int}-1$, and the transfers of the last two intermediaries $s_{(N^{int}-1)}$and $s_{(N^{int}-2)}$ satisfy two conditions:

$$t_{(N^{int}-1),(N^{int}-2)} = \Delta t_{(N^{int}-1),(N^{int}-2)} + \Delta t_{(N^{int}),(N^{int}-1)} > 1,$$

$$\Delta t_{(N^{int}),(N^{int}-1)} \in [z, 1-z].$$

For equilibria of type (2) to exist, it is required that $z \leq \frac{1}{2}$ and $F_{(i),(i-1)} \leq 1-z$ for sellers $(s_{(2)}, s_{(3)}, ..., s_{(N^{int})})$.

In the equilibria of types (1) and (2), there is exactly one seller in each sub-network of type $G^e(G_0)$ expanding the market; the rest of the sellers free ride.

Proof. A series of results will be used from Appendix C. In this proof, we will only focus on the equilibria with non-zero transfers. We also accept that all sellers turn into type $G^b(G_{sh})$ and there is exactly one seller in each sub-network of type $G^e(G_0)$ paying for the access. Without loss of generality, we say that the set of sellers paying for access is $s_{(2)} = s_2 \in G^e_2, ..., s_{(N)} = s_N \in G^e_N$ and the first N^{int} sellers pay a non-zero transfer.

First, any two sellers transferring a positive amount cannot get access from the same seller. To prove it, consider the opposite—then the first seller has incentive to deviate to a contract with the second seller. As a result, sellers that pay a positive transfer form a sequence of intermediaries, such that each seller transfers information to the next seller in the sequence.

Second, suppose s_k gets access from s_{k-1} for transfer $t_{k,k-1} > 0$. For the transfer to be stable we require that s_k does not get a direct profit from reselling:

$$0 < \Delta t_k = t_{k,k-1} - \sum_{l=1}^{N} t_{l,k} \leq 1 - z \text{ for } k = 2, ..., K-1$$

Otherwise, seller s_{k+1}, who gets access from s_k, would "jump over" s_k and cooperate with s_{k-1} at a price below $t_{k+1,k}$.

We further control for the incentives of sellers s_{k-1} and s_{k+1} to deviate to a transfer $\hat{t}_{t+1,t-1} = t_{t+1,t} - \varepsilon$ apart from s_k. If $t_{k-1,k} > 1$, s_{k-1} will not deviate only if

$$\Delta t_k \geq F_{k,k-1},$$

If $t_{k-1,k} \leq 1$, it is not possible to dismiss the deviation unless $k = N^{int}$. As a result, we require $t_{k-1,k} > 1$ for $k < N^{int}$. Therefore, in the equilibrium, seller s_k receives either transfer $t_{k+1,k} \geq z$ or $t_{k+1,k} = 0$. Conditions $z \leq \frac{1}{2}$ and $F_{(i),(i-1)} \leq 1 - z$ become necessary for the non-zero transfers to be stable. ► □

Theorem 14 claims that when sharing does not create negative externalities for sellers, the price of access can be positive only when sellers have less bargaining power than buyers and sellers are not obligated to pay more than their net transfer in the case of default, even if their total transfer is significantly larger.

We have considered the case where s_1 is the only seller who initially trades at the price of one. If initially there are other sellers of type $G^b(G_0)$, equilibria of type (1) are the only possible equilibria. The proof of this fact is straightforward and can be done by contradiction: for any positive transfer, there is a deviation to a smaller positive transfer. Therefore, to avoid deviations, the equilibrium transfers need to be zero.

Appendix E: Proof of Theorem 4

To see why the proposed outcome is an equilibrium, assume that sellers from different subnetworks get access to other subnetworks $G^b(G_0)$ such that all subnetworks are absorbed into one large subnetwork of type $G^b(G_{sh})$ with a single non-trading buyer. The remaining non-trading buyers are shared with the initially weak and balanced sellers. In the equilibrium, the price of sharing reaches minimum $t_{i,j} = 0$. Because the cost of sharing is zero for strong sellers and the information rights are non-transferable, the only transfers that are not blocked by other coalitions are zero transfers. The trade volume generated by the equilibrium network is $m - 1$.

Besides the described equilibrium, there are other stable transfers that exist. First, there is an improving path to the set of networks with at most one non-trading buyer in each subnetwork $G^b(G_0)$. It follows directly from the fact that if there are two non-trading buyers in one of the subnetworks $G^b(G_0)$, access to one of the buyers can be sold to a non-trading seller from $G^s(G_0)$. This type of networks is the other candidate for SPPNS. This outcome does not evolve into the equilibrium that we proposed earlier because of the coordination failure. To see this, consider two arbitrary sellers $G^b(G_0)$ who have exactly one non-trading buyer in each subnetwork. These sellers could merge two networks and sell access to one of the non-trading buyers. In order for one of them to communicate with two different sellers, two pairwise contracts are needed. Therefore, there may be a coordination failure when $z < \frac{1}{2}$. Although we give credit to these outcomes, we still do not consider them as the equilibrium outcomes because they generate total trade volume of strictly less than $m - 1$. ►

Appendix F: Proof of Theorem 5

The basin of attraction is such that sellers from different subnetworks $G^b(G_0)$ and $G^v(G_0)$ combine into one subnetwork $G^b(G_{sh})$ and share all non-trading buyers, but one with sellers from $G^s(G_0)$. We now prove that from any network outside of this set, there is an improving path that leads inside the set, and any deviation from the basin makes at least one seller worse off.

If $s_1 \in G^b(G_{sh})$ and $s_2 \in G^c(G_{sh})$ there exists a pairwise deviation of (s_1, s_2) such that both sellers turn to type G^b. To eliminate similar deviations we consider only networks with subnetworks of types $G^s(G_{sh})$ and $G^b(G_{sh})$ or $G^s(G_{sh})$ and $G^c(G_{sh})$.

a) Suppose set $G^b(G_{sh})$ is non-empty. Then, regardless of the contracts of $s_1 \in G^b(G_{sh})$, s_1 does not have incentives to pairwise deviate and change his type to G^w, because the deviation changes the total payoff of the coalition by at most $1 - 2z \leq 0$. Therefore, we consider the equilibrium candidate G_{sh}, where sellers of type $G^b(G_{sh})$ share access until they have one non-trading buyer in each subnetwork. If sellers have at least two non-trading buyers in two subnetworks, there is always a two-step improving path, where sellers merge in one subnetwork and sell access to $G^s(G_0)$.

b) Consider an equilibrium candidate G_{sh} with only subnetworks of types $G^c(G_{sh})$ and $G^s(G_{sh})$. In this case, a seller of type $G^c(G_{sh}) \cap G^b(G_0)$ who shares access with sellers from $G^s(G_0)$ has incentives to breach one of the sharing contracts and deviate to G^b: $1 > 2z$.

Based on the properties described above, the set described in the first paragraph is the only candidate for being the basin of attraction, because improving paths lead to the basin but not outside the basin. Because the basin always exists, without the specification of transfers, we can say that the statement of the theorem is true.▶

Appendix G: Proof of Theorem 6

Using the same argument as in Appendix F, we consider only networks with subnetworks of types $G^s(G_{sh})$ and $G^b(G_{sh})$ or $G^s(G_{sh})$ and $G^c(G_{sh})$.

a) Suppose set $G^b(G_{sh})$ is non-empty. Then, set $G^s(G_{sh})$ is also non-empty because $m < n$. Among sellers of type $G^b(G_{sh})$, we pick seller s_1, who gets utility that is less than or equal to one. This seller always exists because the total payoff of sellers in $G^b(G_{sh})$ is limited by the benefits they extract from trade—one unit per trader. Then, seller $s_3 \in G^s(G_{sh})$ agrees to deviate to the contract with transfer $\tilde{t}_{3,1} = z$, even if it turns seller s_1 into type G^w. Seller s_3 is neither gaining nor losing from this deviation because the deviation redistributes all gains from trade to s_1. Seller s_1 strictly benefits from this deviation because he gains:

$$z + \tilde{t}_{3,1} = 2z > 1.$$

This proves that stable subnetworks of type $G^b(G_{sh})$ do not exist when we consider $z > \frac{2}{3} > \frac{1}{2}$.

b) Consider an equilibrium candidate G_{sh} where only subnetworks of types $G^w(G_{sh})$ and $G^s(G_{sh})$ are possible. It is intuitively apparent that when the bargaining power of sellers is sufficiently large, sellers of type $G^b(G_0)$ will change type to $G^c(G_{sh})$. It is also clear that sellers of type $G^c(G_0)$ do not change their type to $G^s(G_{sh})$ because in this case, all sellers of new subnetwork gain zero. To show that there is a stable equilibrium of this kind, we determine the equilibrium transfers that eliminate incentives to deviate.

A sharing seller $G^b(G_0) \cap G^w(G_{sh})$ and a weak seller $G^s(G_0)$ have incentives to share and not breach if the transfer between them t_{ij} satisfies

$$1 + (1 - z) - z \leq t_{i,j} \leq z \ ,$$

which simplifies to

$$2(1 - z) \leq t_{i,j} \leq z,$$

$$z > \frac{2}{3}.$$

This requirement eliminates the incentives of seller $G^b(G_0)$ to break the agreement with the seller of type $G^s(G_0)$ and sign a new agreement with a seller of type $G^c(G_{sh})$. Under this condition, all other sellers also do not have incentives to deviate.

The equilibrium network structure is important in explaining the negative externalities that may occur when a seller shares access to a buyer who is connected to another seller. If negative externalities do not exist, sellers charge maximum sharing rents which are $t_{i,j} = z$; this eliminates incentives to lower the price. The status quo networks without negative externalities are characterized by the feature that all subnetworks of type $G^b(G_0)$ have only one seller in them. In the presence of negative externalities (multiple sellers $G^b(G_0)$ in one subnetwork), sellers affected by negative externalities will try to underbid others unless the sharing rent is zero. However, the equilibrium transfer cannot be zero because then the sharing seller would prefer not to share. This paradox shows that a stable equilibrium does not exist when there are multiple sellers in one subnetwork of type $G^b(G_0)$. ▶

Appendix H: Proof of Theorem 7

We first prove that for a pair of a passive/active buyers $b_j \in N_P(G_{sh}) \cup N_A(G_{sh})$ and a non-trading buyer $b_i \in N_N(G_{sh})$ with values $v(b_i) < v(b_j)$ there is a pairwise deviation to network G'_{sh}, such that $b_j \in N_N(G'_{sh})$ and $b_i \in N_P(G'_{sh})$.

Suppose there exists a pair of buyers $b_i \in N_N$ and $b_j \in N_P$ such that $v(b_i) > v(b_j)$. Assume buyer b_j provides the best outside option to some seller s_j. Buyer b_i does not affect on the equilibrium prices but it is connected to seller s_i. Then, there is a profitable pairwise deviation for coalition (s_i, s_j) because sharing access to b_i with s_j does not affect the market power of any seller besides s_j, while it still increases the payoff of s_j by pushing the price up.

If $b_i \in N_N$ and $b_j \in N_A$, such that $v(b_i) > v(b_j)$, then s_i has incentives to sell access to s_j at a very small price. It means there is a deviation which strictly benefits both sellers.

Therefore, to prevent these kinds of deviations in the equilibrium, it is necessary that passive and active buyers value goods more than non-trading buyers. It proves the first part of the theorem.

If any network in the basin of attraction contains a pair of buyers $b_i \in N_N$ and $b_j \in N_P \cup N_A$ such that $v(b_i) > v(b_j)$, there is always a path of deviations that will reallocate the buyers as we described above. This path will eventually lead to the network where all non-trading buyers value goods less than the rest of buyers. We proved the second statement of the theorem by contradiction.▶

Appendix I: Proof of Theorem 8

We have already shown that any non-trading buyer values goods less than any passive buyer. Therefore, we only need to prove that any passive buyer values goods lower than any active buyer. When we assume the opposite, there exists a passive buyer $b_i \in N_P(G_{sh})$ and an active buyer $b_j \in N_A(G_{sh})$, such that $v(b_i) > v(b_j)$. Without loss of generality, we assume that buyer b_i provides an outside option for s_i and trades with s_k at price

$$p_{i,k} = (1-z)\underline{p_{i,k}} + z\overline{p_{i,k}},$$

and buyer b_j trades with s_j at price

$$p_{j,j} = (1-z)\underline{p_{j,j}} + z\overline{p_{j,j}}.$$

Then, the profit of s_i from the trade can be expressed as

$$u(s_i) = (1-z)\underline{p_{i,k}} + z\overline{p_{i,i}} - v(s_i).$$

We consider a deviation when s_i connects seller s_j with buyer b_i. Using the mechanism of Elliott (2014) for finding alternative options, we can calculate that the trade profit of s_i changes from

$$u(s_i) = (1-z)v(b_i) + z\overline{p_{i,k}} - v(s_i),$$

to

$$u^{new}(s_i) \geq (1-z)\overline{p_{j,j}} + z\overline{p_{i,k}} - v(s_i),$$

which results in profit changes

$$\Delta u(s_i) \geq (1-z)(\overline{p_{j,j}} - v(b_i)) \geq (1-z)(v(b_j) - v(b_i)).$$

The profit of s_j changes as follows:

$$\Delta u(s_j) = (1-z)(\underline{u}^{new}(s_j) - \underline{u}(s_j)) + z(v(b_i) - v(b_j)).$$

As a result, the joint utility of sellers increases by

$$\Delta u(s_i) + \Delta u(s_j) \geq (2z-1)(v(b_i) - v(b_j)) > 0.$$

We have proved that there is a pairwise deviation that benefits both sellers. Similar to the proof of Theorem 7, we reach a contradiction that proves Theorem 8.▶

Appendix J: Proof of Theorem 9

Suppose market trade volume in G_{sh} is below the Walrasian level. Then, without loss of generality, there are two non-trading traders, s_2 and b_1, such that $v(b_1) > v(s_2)$. Seller s_2 has incentives to get access to b_1, which will change his utility by $\Delta u(s_1) = u^{new}(s_1) - u(s_1)$. Assume that seller in s_1 is connected to buyer b_1.

If b_1 does not serve as an outside option, it is straightforward that seller s_1 connected to b_1 benefits from sharing with s_2.

If b_1 serves as an outside option for s_1 and $q_{1,j} > 0$, then

$$u(s_1) = (1-z)(v(b_1) - v(s_1)) + z(\alpha_{1,j} - \underline{u}(b_j)),$$

$$u^{new}(s_1) = (1-z)(v(s_2) - v(s_1)) + z(\alpha_{1,j} - \underline{u}(b_j)),$$

and

$$\Delta u(s_1) = (1-z)(v(s_2) - v(b_1)).$$

The utility of s_2 changes by amount

$$\Delta u(s_2) = z(v(b_1) - v(s_2)).$$

As a result, the joint utility of sellers increases by

$$\Delta u(s_1) + \Delta u(s_2) = (2z-1)(v(b_1) - v(s_2)) > 0.$$

By contradiction, we conclude that G_{sh} is not stable. Therefore, to prevent these kinds of deviations in the equilibrium, it is necessary that the number of trades is greater than or equal to Q^{walr}. From Theorem 8 it follows that the set of trading buyers should include the set of buyers who trade in the Walrasian outcome.

If network G_{sh} is in the basin of attraction that satisfies the properties described in Theorem 8 and the market trade volume falls below the Walrasian level, there is a path of deviations that will reallocate the buyers as we described above. This path will eventually lead to the network where market volume weakly exceeds Q^{walr}. We proved the second part of the theorem.▶

Appendix K: Proof of Theorem 10

Suppose, in the equilibrium network G_{sh}, there exists a subnetwork $G^c(G_{sh})$ and a subnetwork $G^b(G_{sh})$ with at least two non-trading buyers. In this case, there is a deviation by a balanced seller from $G^c(G_{sh})$ and a strong seller from $G^b(G_{sh})$ that makes both sellers strong. Therefore, we exclude these networks G_{sh} from the equilibrium candidates.

Consider an equilibrium candidate G_{sh} with a seller of type $s_i \in G^c(G_{sh}) \cap G^b(G_0)$ who sold the market access to another seller $s_j \in G^b(G_{sh}) \cap G^c(G_0)$. In this case, s_j needs to compensate s_i by the amount greater

than $1-z$. However, it is not possible, because s_j gets a profit increase in the amount of $1-z$ and other sellers connected to s_j do not compensate s_j for positive externalities. Therefore, all sellers of type $G^b(G_0)$ do not benefit from becoming of type $G^c(G_{sh})$.

Suppose there are $|G^b(G_0)|$ strong-seller subnetworks of type $G^b(G_0)$. Then, there are at least $|G^b(G_0)|$ buyers who do not trade and belong to $G^b(G_0)$. Therefore, the number of balanced subnetworks that benefit from the information sharing is at least $m-n-|G^b(G_0)|$. In fact, there can be more of them, if at least two sellers $G^b(G_0)$ share access to the same buyer with two different sellers from $G^c(G_0)$. In this case, the strong sellers do not impose negative externalities on each other, because the balanced sellers get market access only to increase their prices and not trade volume. Therefore, the information about the same buyer can be sold to two different sellers.

The stable transfers that support this equilibrium are as follows: if set $G^c(G_{sh})$ is non-empthy, the information is shared at $t=1-z$. If set $G^c(G_{sh})$ is empty, the information is shared for free.▶

Appendix L: Proof of Theorem 12

We need to prove that, in the basin of attraction, each subnetwork with initially strong sellers leaves one non-trading buyer in the subnetwork and shares access to the rest of the non-trading buyers with initially balanced and weak sellers, preferring balanced sellers. It means we need to prove that from any network outside of this set, there is an improving path that leads inside the set and any deviation from the basin makes at least one seller worse off.

a) Suppose set $G^b(G_{sh})$ is non-empty. Then, regardless of the contracts of $s_1 \in G^b(G_{sh})$, s_1 does not have incentives to pairwise deviate and change his type to G^c, because the deviation changes the total payoff of the coalition by at most $1-2z \leq 0$. Therefore, we consider the equilibrium candidate G_{sh} where sellers of type $G^b(G_{sh})$ share access until they have one non-trading buyer in each subnetwork. If sellers have at least two non-trading buyers in two subnetworks, there is never an improving path, where sellers merge in one subnetwork and sell access to $G^s(G_0)$ because it is not allowed by our network formation rules.

b) Consider an equilibrium candidate G_{sh} with only subnetworks of types $G^c(G_{sh})$ and $G^s(G_{sh})$. Then a seller of type $G^c(G_{sh}) \cap G^b(G_0)$ who shares access with sellers from $G^s(G_0)$ has incentives to breach one of the sharing contracts and deviate to G^b: $1>2z$.

c) The non-trading buyer connected to a strong seller can be sold to a weak seller $G^s(G_0)$ or to a balanced seller $G^c(G_0)$. It is more beneficial for strong sellers to support balanced sellers when they can get a higher price from this sharing: $z<1-z$ because we consider the case when $z<1/2$. When the number of buyers is insufficient to provide each seller with a pair of buyers, strong sellers support balanced sellers first. It means that information sharing with transferable rights leaves the prices of all initially strong sellers at $p^s=1$, changes the prices of at least $m-n-|G^b(G_0)|$) balanced and weak sellers to $p^s=1$, and the price of at most one balanced seller to $p^s=z$. The rest of prices stay at either at $p^s=1$ or $p^s=0$, depending on the status quo prices. Therefore, we can calculate the total increase in the volume of trade as the number of initially weak sellers that increased their volume of trade. Since there are more sellers than buyers, more than one

buyer will be unmatched with a potential seller because of the lack of structural changes in the network with information sharing rights.

Based on the properties described above, the set described in the first paragraph is the only candidate for being the basin of attraction, because improving paths lead to the basin but not outside the basin. Because the basin always exists, without the specification of transfers, we can say that the statement of the theorem is true.►

Appendix L: Proof of Theorem 13

The proof of the theorem is identical to the one in Appendix *G*.